

## **Title: Forces on an Inclined Plane**

An interdisciplinary lesson investigating the force on an incline plane and the development of a mathematical model.

### **Link to Outcomes:**

- **Problem Solving** Students will demonstrate the ability to solve problems from within and outside of mathematics with applications to real-world situations.
- **Communication** Students will demonstrate the ability to communicate mathematics with language and the symbolism of mathematics.
- **Connections** Students will demonstrate the ability to make connections among various mathematical topics and the applications to other disciplines.
- **Technology** Students will demonstrate the ability to use technology where appropriate as they solve real world problems.
- **Cooperation** Students will demonstrate the ability to solve mathematics problems in a cooperative atmosphere.
- **Trigonometry** Students will demonstrate the ability to apply trigonometry to problem-solving situations with triangles, and to explore real world phenomena using the sine and cosine functions.
- **Statistics** Students will demonstrate the ability to draw inferences about data and to transform data to aid in interpretation and prediction to test hypotheses using statistics.
- **Concepts of Science** Students will demonstrate their acquisition and integration of major concepts and unifying themes from physical science.
- **Nature of Science** Students will demonstrate the ability to interpret and explain information generated by their exploration of scientific phenomena.
- **Habits of Mind** Students will demonstrate ways of thinking and acting inherent in the practice of science.
- **Processes of Science** Students will demonstrate the ability to employ the language, instruments, methods and materials of science for collecting, organizing, interpreting, and communicating information.

**Brief Overview:**

When an object is on an inclined plane, the force of gravity on the object is broken up into two component forces. One component force is perpendicular to the inclined plane and the other is parallel to it. In this activity, students will measure the parallel force at various angles of elevation and study the relationship that exists between the size of the parallel force and the angle of elevation.

**Grade/Level:**

Grades 10–12, Trigonometry, Functions, Physics

**Duration/Length:**

2–3 class periods (45 minutes each)

**Prerequisite Knowledge:**

- Knowledge of regression equations and correlation coefficients
- Familiarity with quadratic, sine, and cosine functions
- Familiarity with the statistical features of a graphing calculator
- Familiarity with methods for measuring force
- Knowledge of appropriate metric units of force

**Objectives:**

Students will

- Test the accuracy of their trigonometric model on the TI-82 graphics calculator.
- Use the statistics mode of the TI-82 graphics calculator to construct a scatter plot and interpret the set of data points.
- Observe a set of data that when graphed suggests a pattern that could represent a function.
- Calibrate a force probe and use it to take measurements using the Calculator Based Laboratory (CBL).

**Materials/Resources/Printed Materials:**

- Inclined plane
- Motion cart
- Ring stand and clamps
- Standard weights (100 g, 500 g, 1 kg)
- Rubber bands or tape
- String or wire
- Chalkboard protractor
- CBL unit
- Force measuring probe
- DIN adapter
- Graphing calculator (calculator instructions are written for the TI-82 graphics calculator)
- Included worksheets

**Development/Procedures:**

Calibrate a Vernier force probe for use with the CBL.

Use the force probe to collect force data for an object on an inclined plane at various angles of elevation.

Develop a calibration formula and use the formula to convert voltage readings into newtons using the TI-82 graphics calculator.

Use the TI-82 to model and interpret graphed experimental data.

**Evaluation:**

Students will demonstrate correct recording and reporting of data on science lab sheet.

Students will demonstrate correct modeling of the recorded data in the mathematics classroom.

**Extension/Follow Up:**

Measure the static coefficient of friction on the inclined plane at various angles.

Write a TI-82 program to calibrate the force probe, derive the calibration equation, and convert voltage readings to force.

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## **Forces on an Inclined Plane Laboratory Activity**

### **DISCUSSION**

When an object is on a hill or an inclined plane, the gravitational force acting on the object (the weight of the object) is broken into two component forces. One force is perpendicular to the inclined plane and holds the object onto the surface, and the other force is parallel to the inclined surface and pushes the object down the hill. This parallel force has many practical applications including providing the force that propels a roller coaster. In this activity you will investigate changes in the parallel force that occur as the angle of elevation of the inclined plane is changed.

Data will be collected using a force probe with the CBL. You will first need to calibrate the force probe so the voltage data you collect using the CBL can be converted to newtons of force for data processing.

### **MATERIALS**

Inclined plane  
Motion cart  
Ring stand and clamps  
Standard weights (100 g, 500 g, 1 kg)  
Rubber bands or tape  
String or wire  
Chalkboard protractor  
CBL unit  
Force measuring probe  
DIN adapter  
Graphing calculator (calculator instructions are written for the TI-82 graphics calculator)  
Included worksheets

### **PROCEDURE**

#### **A. Calibration of the force probe**

1. Clamp the force probe to a ring stand with the sensor arm in a horizontal position.
2. Connect the force probe to the CBL using the DIN adapter.
3. Turn the CBL on and press {MODE} so the meter indicates "sampling."
4. Record the voltage value from the CBL with no mass attached to the probe.
5. Suspend a 0.980 N (100 g) weight from the probe. Record the voltage reading from the CBL. The conversion factor is 9.8 Newtons per kilogram. This should be used if other weights are used.
6. Repeat using the 4.90 N (500 g), and the 9.80 N (1 kg) weights.
7. Enter the calibration data values in the TI-82 calculator. (The voltage readings should be entered in List 1, and the force (weight) values should be entered in List 2.)
8. Plot the calibration data.

9. Use the data plot to determine the equation of a best fit line (linear regression) for the data. (See TI-82 Instructions.)
10. Record the equation in your lab report and enter the equation as  $y_1 =$  on the TI-82. (This equation will be used to convert probe voltages into newtons of force in the data treatment part of this activity)

#### B. Parallel force vs. angle of elevation – Data collection

1. Clamp the force probe to a ring stand with the sensor arm in a horizontal position.
2. Read the voltage value from the CBL with no mass attached to the probe. This value should be recorded at  $0^\circ$  angle of elevation on the data table.
3. Fasten a 4.90 N (500 g) weight to the motion cart using tape or rubber bands.
4. Hang the cart on the hook of the sensor arm of the force probe and read the voltage value from the CBL. This value should be recorded at  $90^\circ$  angle of elevation on the data table. This value will give the total weight (or force of gravity) of the cart and weight together.
5. Position the ramp at  $5^\circ$  angle of elevation.
6. Connect the cart to the hook on the sensor arm of the force probe and position the cart on the ramp.
7. Hold the force probe against the top end of the ramp with the sensor arm perpendicular to the surface of the ramp. Hold the housing of the probe against the ramp. The sensor arm should not touch the ramp.
8. Read the voltage value and record the value on the data table.
9. Repeat steps 6, 7, and 8 with the ramp positioned at  $15^\circ$ ,  $25^\circ$ ,  $35^\circ$ , ...,  $85^\circ$ . Record the voltage value at each angle of elevation in the data table.
10. Enter all values from the data table in the TI-82. Angle measurements should be entered in List 1, and the probe voltage measurements entered in List 3.

#### C. Modeling the data using the TI-82

1. Convert force data from voltage readings to Newtons.
  - a. Highlight List 2 (column heading): Press **[STAT]**, **[ENTER]** to edit. Press **[ $\Delta$ ]** and **[ $\nabla$ ]**.
  - b. Convert data and place in List 2: Press **[Y-VARS]**, **[ENTER]**, **[ENTER]**. Press **[ $\square$ ]**, **[L3]**, **[ $\square$ ]**, **[ENTER]**.
2. Graph the data in List 1 and List 2. \*
3. Find the line of best fit for quadratic. \*
4. Check accuracy of line of best fit using residuals.
  - a. Highlight List 3: Press **[STAT]**, **[ENTER]**, **[ $\Delta$ ]**, **[ $\nabla$ ]**, **[ $\nabla$ ]**.
  - b. Clear List 3: Press **[CLEAR]**, **[ENTER]**.
  - c. Enter the value of the square root of List 2 into List 3: Press **[ $\Delta$ ]**, **[2nd]** **[ $\sqrt{\phantom{x}}$ ]**, **[2nd]** **[L2]**, **[ENTER]**.
5. Graph the data in List 1 and List 3. \*

\* See TI-82 Instructions.

6. Expand the graph to have a range from  $0^\circ$  to  $270^\circ$ .
  - a. Clear List 3, List 4, List 5, and List 6.
  - b. Enter angle measures from  $95^\circ$  -  $180^\circ$  in List 3: Press [STAT], [ENTER],  $\Delta$ ,  $\triangleright$ ,  $\triangleright$ . Press [2nd] [L1] + 90, [ENTER].
  - c. Enter data from List 2 into List 4 in reverse order: Press [STAT], [ENTER],  $\Delta$ ,  $\triangleright$ ,  $\triangleright$ ,  $\triangleright$ . Press [2nd] [L2], [ENTER]. Press [STAT], [SORTD], [2nd] [L2],  $\square$ , [ENTER].
  - d. Enter angle measures from  $185^\circ$  -  $270^\circ$  in List 5. Press [STAT], [ENTER],  $\Delta$ ,  $\triangleright$ ,  $\triangleright$ ,  $\triangleright$ ,  $\triangleright$ . Press [2nd] [L3] + 90, [ENTER].
  - e. Enter the negative value for the data in List 2 into List 6. Press [STAT], [ENTER],  $\Delta$ ,  $\triangleright$ ,  $\triangleright$ ,  $\triangleright$ ,  $\triangleright$ ,  $\triangleright$ . Press [ $\ominus$ ], [2nd] [L2], [ENTER].
  - f. Record data values for  $90^\circ$  and  $270^\circ$ . We will need these values for ACOS program.
7. Graph all the data. (Plot 1 will graph List 1 and List 2, Plot 2 will graph List 3 and List 4, and Plot 3 will graph List 5 and List 6) \*
8. Find the line of best fit using the ACOS program.
  - a. Link graphing calculators to receive the ACOS program. \*
  - b. Execute ACOS program. (Use recorded maximum and minimum values to check the accuracy of the trace in the program.)

\* See TI-82 Instructions

## LAB WORKSHEET

## Calibration Table

CBL voltage ( $L_1$ )	Weight (N) ( $L_2$ )

### calibration equation

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### Angle of elevation vs parallel force data

[illegible]

## Teacher Notes:

### MATERIALS

Inclined plane: A commercial inclined plane apparatus can be used, or a 1" x 6" board about 0.5 m long will work just as well. Some inclined plane apparatuses stop at  $45^\circ$  angle of elevation; a chalkboard protractor can be used with the ramp manually held for larger angles of elevation.

Motion cart: a Halls cart or any other free rolling cart

String or wire: used to connect the cart to the force probe

Chalkboard protractor: a large demonstration protractor will give more accurate readings than a small student protractor.

CBL and force probe: This lab can be carried out using a spring scale to obtain force readings. The calibration can then be omitted, and the force data from the spring scale used directly in the data analysis. This method will not yield the same "clean" data, but it will still work.

### PROCEDURE

#### Data Collection

Part B - Step 3) A 1.0 kg weight can be used if the motion cart is light, but the TOTAL MASS must be less than 2.0 kg to avoid damaging the force probe.

Part B - Step 9) Take measurements every  $5^\circ$  if time permits. More data points will make the data easier to interpret.

Part B - Step 9) At  $85^\circ$  the cart wheels may lift from the ramp. If this occurs use  $80^\circ$  angle of incidence instead, making sure to enter  $80^\circ$  in the data table instead of  $85^\circ$ .

Part C) Data should be saved in List 1 and List 3. If not, enter data from lab worksheet.

Part C - Step 5) When checking the graph using residuals, the graph is not linear. Therefore the function cannot be a quadratic.

Part C - Step 6) The students will explore the trigonometric model. Students must create data for angles ranging from  $90^\circ$  to  $270^\circ$ . This will be done through an understanding of the symmetry of the sine function.

Part C - Step 8) Teachers will need to have loaded the ACOS program prior to class. Students will receive the program from the teacher's calculator during the linking operation. The ACOS program is a part of the TI-82 instructions.

Execution of the ACOS program will show the following screen:

```
ENTER  
TRACE TO MAX  
ENTER  
TRACE TO MIN  
ENTER
```

Follow these instructions to arrive at the equation.

Record the equation.

Press {ENTER} to see the graph of the data with the curve.

## LAB WORKSHEET (sample data)

### Calibration Table

CBL voltage ( $L_1$ )	Weight (N) ( $L_2$ )
0 N	3.21
0.980 N	3.11
4.90 N	2.74

### calibration equation

$$y = -10.47145187602x + 33.58378466558$$

### Angle of elevation vs parallel force data

[illegible]

**Sample screen:** This displays expanded data.

L 1	L 2	L 3
5	.59871	95
15	2.2741	105
25	3.9496	115
35	5.3109	125
45	6.5674	135
55	7.7193	145
65	8.6617	155
L 1 ( 1 ) = 5		

L 4	L 5	L 6
9.4994	185	-.59871
9.29	195	-2.274
9.1853	205	-3.95
8.6617	215	-5.311
7.7193	225	-6.567
6.5674	235	-7.719
5.3109	245	-8.662
L 6 ( 1 ) = -.5987112...		

## TI-82 INSTRUCTIONS

### PROCEDURE FOR LINKING TI-82 GRAPHICS CALCULATORS<sup>1</sup>

1. **Receiver calculator only.** Clear all lists:

Press [STAT]. Press [4]. Press [2nd] [L1]. Press [□]. Press [2nd] [L2]. Continue until all lists are named and then press [ENTER].

2. Connect the TI-82 Link cable to both calculators at the I/O port located at the bottom edge of the calculator. Make sure the cable ends are **flush** to the calculator. If the cable is not connected correctly, the sender screen will show the message "Error in Xmit" when the sender tries to transmit.
3. Press [2nd] [LINK] on both calculators.

#### **Receiver directions:**

Press [1]. RECEIVE will be highlighted. Press [ENTER]. The screen will show the message "Waiting...." Once the lists have been transmitted, the screen will display the message "Receiving..." and name lists received followed by the message "Done."

#### **Sender directions:**

Choose option 2: **Select All** - Press [✓] to L2. Press [ENTER]. Press [✓] to L3. Press [ENTER]. Press [✓]. You will observe boxes in front of the lists chosen to be transmitted. Press [2]. Transmit will be highlighted. Press [ENTER]. The screen will display the names of the transmitted lists.

4. Remove the TI-82 link cable. Students may now link the data to other students' calculators.

**Important !!!** Procedure 1. (above) should not be repeated by any student who is a sender. Only receivers need to clear lists.

5. Press [STAT]. Press [ENTER]. The students will now have the collected data points in lists L2 and L3.

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<sup>1</sup> NSA SUMMER INSTITUTE MATHEMATICS NOTEBOOK, 1994

## Using Statistical Functions on the TI-82

### Entering the data:

1. Clear all previous data

Press [STAT]; Press [4] and then press [2nd] [L1] [2nd] [L2] [ENTER]  
(this should clear all of the data in these two lists)

2. To enter edit mode

Press [STAT] and press [1] (to select Edit..)

3. Entering the data

Type in the value for the first value of x in the L1 position. Press [ENTER]. This will set you up to enter the next x value. Using the [▸] key will allow you to enter the corresponding y values.

4. To get back to the main screen

Press [2nd] [QUIT]

### Graphing the data:

1. Turn off all currently operating graphs

Press [2nd] [STAT PLOT]. Press [4] and [ENTER].

2. To enter statistical graphing mode and set up parameters

Press [2nd] [STAT PLOT]. Press [1]. Page down through the selection using the [▾] key. Change the screen so that the following selections are highlighted.

ON

Type: [ . . . ]

Xlist: L1

Ylist: L2

Mark: □

[Note: when simultaneously graphing you may choose to use different marks.]

3. To graph the data

Press [ZOOM] [9].

This command automatically adjusts the window for statistical data.

**Line of best fit:** When calculating the line of best fit it is important that you are evaluating the right data lists. To verify this you can check the 2- Var Stats under the SetUp command by pressing [STAT] and [2] to Calc.

1. Linear equations  $y = ax + b$

A. Calculating the equation for the regression line

Press [STAT]. Press [2] to Calc. Use the [2] to get to 5 and press [ENTER] twice.

B. Setting up the equation in the Y = editor

Press [Y=]. The following steps will allow you to set up the actual equation.

Press [VARS]. Press [5]. Press the [2] twice to get to EQ. Press [7].

C. Graphing the regression equation on the data

Press [GRAPH].

2. Quadratic equations  $y = ax^2 + bx + c$

A. Calculating the equation for the regression line

Press [STAT]. Press [2] to Calc. Use the [2] to get to 6 and press [ENTER] twice.

B. Setting up the equation in the Y = editor

Press [Y=]. The following steps will allow you to set up the actual equation.

Press [VARS]. Press [5]. Press the [2] twice to get to EQ. Press [7].

C. Graphing the regression equation on the data

Press [GRAPH].

## ACOS program<sup>2</sup>

```
ClrHome
Disp "ENTER"
Disp "TRACE TO MAX"
Disp "ENTER"
Disp "TRACE TO MIN"
Disp "ENTER"
Pause
Input
X C:Y J
Input
X S:Y T
/(S-C) B
(T+J)/2 D
J-D A
ClrHome
Disp "Y="
Disp "Acos (B(X-C))+D"
Disp "{A,B,C,D}="
Disp A,B,C,D
Pause
"Acos (B(X-C))+D" Y1
DispGraph
```

<sup>1</sup> NSA SUMMER INSTITUTE MATHEMATICS NOTEBOOK, 1994

<sup>2</sup> John Diehl, Hinsdale Central High School, Hinsdale, IL  
Presented at NCTM National Conference, April 1995